Name:_____

SM3 4.6 Exponential & Log Problems

Exponential and logarithmic word problems are not too challenging to set up. They will typically take one of four forms based on their classification as growth or decay and their classification as continuous or per unit of time.

	Growth	Decay	
Continuous	$y = ae^{rt}$	$y = ae^{-rt}$	The models for growth and decay are almost identical. The only difference is a – sign in front of r.
Per unit of time	$y = a(1 + r)^t$	$y = a(1 - r)^t$	

Variable	What the variable represents
y	The amount at time t
a	The amount at time 0; y_0
r	The rate of change
t	The time
e	e is not a variable! $e \approx 2.71828$ … always!

Setting up equations from a paragraph of information is easy:

Step 1) Determine if the problem represents growth or decay. Most growth problems will say the word grow or growth, or words that mean growth like appreciate, increase, or develop.

Step 2) Determine if the problem needs a continuous model or per unit of time. Spotting a "per unit of time" normally means looking for a phrasing that includes how often something is being measured (e.g., girlfriends per year, girlfriends per minute, etc.). Unfortunately, there are many synonyms for per (e.g., girlfriends a year, girlfriends each minute, etc.). If this phrasing is not included, you may assume a continuous model will be used. Radioactive decay chemistry problems will always be handled continuously.

Step 3) Select the right formula from the table above. You'll have to have these memorized.

Step 4) Determine the value of the above variables, including which one you must solve for, and substitute them into the equation you built in the previous step.

Step 5) Use your equation-solving techniques to find the value of the unknown variable. You'll likely need a calculator to do some of the evaluating for you.

Step 6) Write a sentence that includes what you've found within the context of the problem.

Example: If $\$1500$ is invested and after 7 years you have $\$2830$, what is the interest rate, if it is compounded continuously?

Step 1) The money amount went up. This must be a model.
Step 2) The word " " makes me think this should be handled .

$y =$ Step 3) Pick the right equation from the table.

Step 4) Plug in values from the problem.	Variable	Helpful phrase from problem	Value of variable
	y	after 7 years you have $\$2830$	$y =$
	a	$\$1500$ is invested	$a =$
	r	what is the interest rate	$r =$
	t	after 7 years	$t =$

You don't need to make a fancy table like I did here. Your goal is to plug this information into the equation from Step 3), so once you get the hang of it, Step 4) should be a mental exercise that easily flows from Step 3).

$$2830 = 1500e^{7r}$$ Step 4) Plug in values from the problem.

$$\frac{2830}{1500} = e^{7r}$$ Division

$$\ln\left(\frac{2830}{1500}\right) = 7r$$ ln( ) both sides

$$\frac{\ln\left(\frac{2830}{1500}\right)}{7} = r$$ Division

$$0.09 \approx r$$ Use a calculator

Step 5)

Step 6) The interest rate is about .

Mr. Sinister has infected 6 of the X-men with his vampiric virus on October 1st. Each of those X-men infects the people around them at a rate modeled by the equation $y = ae^{0.012t}$, where t is measured in days.

1. Is this a growth or decay model?

2. What is the rate of growth or decay?

3. What is the initial number of infected people?

4. What is the time, *t*, in the calculation of people infected in one month's time?

5. What is the formula used to solve the problem?

6. How many people will be infected in one month's time?

Beast has discovered a cure for Mr. Sinister's virus. It is an electro-magnetic pulse that travels from his location outward. Beast sets off the pulse on November 1st. The pulse instantly cures Beast, who was infected, and also cures the other people who have been effected at a rate modeled by $y = ae^{0.052t}$, where t is measured in days. The pulse also halts any further spread of the infection the moment it fires.

7. Is this a growth or decay model?

8. What is the rate of growth or decay?

9. What is the number of cured people when Beast begins the electro-magnetic pulse?

10. How many people could Beast cure using the pulse during the month of November?

11. If Beast begins the pulse on Nov 1st, how long until the infection be completely cured?

13. Tiger Industries bought a computer for $2500. It is expected to depreciate at a rate of 20% per year. What will the value of the computer be in 2 years?

14. The Martins bought a condominium for $85000. Assuming that the value of the condo will appreciate at most 5% a year, how much will the condo be worth in 5 years?

Bacteria usually reproduce by a process known as binary fission. In this type of reproduction, one bacterium divides, forming two bacteria. Under ideal conditions, some bacteria reproduce every 20 minutes.

15. Find the constant *r* for this type of bacteria under ideal conditions.

16. Write the equation for modeling the exponential growth of this bacterium.

The annual Gross Domestic Product of a country is the value of all of the goods and services produced in the country during a year. During the period 1995-2009, the Gross Domestic Product of the United State grew about 3.2% per year, measured in 2006 dollars. In 1995, the GDP was $5717 Billion.

17. Assuming this rate of growth continues, what will the GDP of the United States be in 2020?

18. In what year will the GDP reach $20 trillion?

19. In 1928, when the high jump was first introduced as a women's sport at the Olympic Games, the winning women's jump was 62.5 inches, while the winning men's jump was 76.5 inches. Since then, the winning jump for women has increased by 0.38% per year, while the winning jump for men has increased at a slower rate, 0.3%. If these rates continue, when will the women's winning high jump be higher than the men's?

20. The Brutus family bought a new house 10 years ago for $120,000. The house is now worth $191,000. Assuming a steady rate of growth, what was the yearly rate of appreciation

Compound interest

21. If $2000 is invested with an annual rate of 6%, compounded annually, how long will it take the money to double? How much money will you have after 10 years?

22. If $2000 is invested and after 5 years you have $2665, what is the interest rate, if it is compounded monthly?

23. If $2000 is invested with an annual rate of 5%, compounded daily, how long will it to increase the amount of money by $400?

24. Find the time necessary for $\$1000$ to double if it is invested at a rate of $r = 7\%$ compounded continuously.